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Relations between directional distribution coefficients in alpha-decay of oriented nuclei

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Abstract. – Angular distributions of α-particles emitted by oriented nuclei are considered theoretically. Exact kinematical relations between the values of directional distribution coefficients have been found for the case when $I_{\mathrm{i}} > I_{\mathrm{f}}$. Another class of the relations is found for the α-decay of compound nuclei in nuclear reactions.

Emission of α-particles in the decay of deformed oriented nuclei, *i.e.*, nuclei with a preferential spin direction in space, should be anisotropic. This was first theoretically predicted by Hill and Wheeler [1] who argued that in a nucleus with a deformed Coulomb barrier, having non-uniform barrier height and width, the tunnelling probability becomes direction dependent. Later theoretical works [2–5] have shown that anisotropy of α-emission is determined not only by the deformed barrier but also by the nuclear structure. Recently, extensive high-resolution measurements of angular distributions in α-decay of oriented odd-mass nuclei have been reported [6–10]. In the experiments, a combination of production by heavy-ion reactions, on-line separation, and on-line implantation and orientation at low temperatures [6] has been successfully applied for studying the α-transitions in odd-mass isotopes of At, Rn [7,11], Bi [8], Pa and Fr [9]. In all cases remarkably pronounced and strongly varying from isotope to isotope α-emission anisotropy has been revealed. The experimental angular distributions have been analyzed by expanding in terms of Legendre polynomials; the coefficients of the expansion have been compared with those calculated in various theoretical models. The comparison reveals that the angular distributions can provide very interesting and important information about both the nuclear structure and the properties of the deformed Coulomb barriers.

Although the dynamics of the anisotropic α-emission is still not completely understood, in this letter we are concerned not with dynamical but rather with kinematical properties of the directional distribution coefficients. We shall show that for some of the transitions these coefficients are not independent but related by simple equations. The derived equations are exact in a sense that they do not depend on the dynamics of the decay and therefore they should be fulfilled in any theoretical model and also in experiment. Thus they may be useful for testing the calculations and for checking the consistency of experimental analysis.

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A general expression for the angular distribution of α-particles emitted from oriented nuclei has been derived by Krane [11] on the basis of a density matrix and statistical tensor formalism. For an ideal detector it may be written in the following form:

$$W(\theta) = 1 + \sum_{k\neq 0} A_k B_k P_k(\cos\theta). \tag{1}$$

Here θ is the emission angle relative to the nuclear orientation direction chosen as z-axis, $P_k(x)$ are the Legendre polynomials, and B_k describe the nuclear orientation. The A_k are the directional distribution coefficients which contain information on the α-transition. They may be written in the form [11]

$$A_k = \frac{\sum_{L,L'} a_L a'_L \cos(\sigma_L - \sigma_{L'}) F_k^\alpha(L, L', I_\mathrm{f}, I_\mathrm{i})}{\sum_L a_L^2}, \tag{2}$$

where I_i and I_f are the nuclear spins in the initial and the final state, respectively, a_L and σ_L are the amplitude and phase of the α-wave with angular momentum L, and F_k^α are the F-coefficients modified for α-decay [11]:

$$F_k^\alpha(L, L', I_\mathrm{f}, I_\mathrm{i}) = (-1)^{I_\mathrm{i}+I_\mathrm{f}+k} \hat{L}\hat{L}'\hat{I}_\mathrm{i}(L0, L'0 \mid k0) \begin{Bmatrix} L & L' & k \\ I_\mathrm{i} & I_\mathrm{i} & I_\mathrm{f} \end{Bmatrix}. \tag{3}$$

Here $(L0, L'0 \mid k0)$ is a Clebsch-Gordan coefficient, $\hat{I} \equiv \sqrt{2I+1}$, and the standard notation for the $6j$ symbol is used. Since parity is conserved in α-decay only even L or only odd L contribute to the sums in (2) depending on parities of the initial and the final nuclear states. The k-values are always even and $k \leq 2I_\mathrm{i}$.

Usually, from the analysis of the experimental angular distributions the coefficients A_k are deduced which are then compared with the theoretical predictions based on some particular model of α-decay. In both experimental and theoretical analysis the directional distribution coefficients A_k are considered as independent. However, as we show below, this is not always true.

Consider the following sum:

$$\sum_{k=0}^{k_\mathrm{max}} (-1)^{I_\mathrm{i}-M_\mathrm{i}} \hat{I}_\mathrm{i}(I_\mathrm{i}M_\mathrm{i}, I_\mathrm{i}-M_\mathrm{i} \mid k0) F_k^\alpha(L, L', I_\mathrm{f}, I_\mathrm{i}) = \hat{I}_\mathrm{i}^2 (I_\mathrm{f}M_\mathrm{f}, I_\mathrm{i}M_\mathrm{i} \mid L0)(I_\mathrm{f}M_\mathrm{f}, I_\mathrm{i}M_\mathrm{i} \mid L'0). \tag{4}$$

The last equation is obtained using the known relations between Clebsch-Gordan and $6j$ coefficients (see eq. (8.7.5.31) in [12]). We note that if $I_\mathrm{i} > I_\mathrm{f}$ then for any $M_\mathrm{i} > I_\mathrm{f}$ the right-hand side of eq. (4) turns to zero independently of L, L' due to the rule of projections in the Clebsch-Gordan coefficients. Therefore, the following equation holds:

$$\sum_{k=0}^{k_\mathrm{max}} (-1)^{I_\mathrm{i}-M_\mathrm{i}} \hat{I}_\mathrm{i}(I_\mathrm{i}M_\mathrm{i}, I_\mathrm{i} - M_\mathrm{i} \mid k0) A_k = 0 \tag{5}$$

or

$$1 + \sum_{k\neq 0} (-1)^{I_\mathrm{i}-M_\mathrm{i}} \hat{I}_\mathrm{i}(I_\mathrm{i}M_\mathrm{i}, I_\mathrm{i} - M_\mathrm{i} \mid k0) A_k = 0 \tag{6}$$

for each $M_\mathrm{i} > I_\mathrm{f}$. The number of such relations is $I_\mathrm{i} - I_\mathrm{f}$. Equation (6) is our final result. It means that if $I_\mathrm{i} > I_\mathrm{f}$ the directional distribution coefficients are not independent. They are related by $I_\mathrm{i} - I_\mathrm{f}$ equations. The equations are exact and do not depend on the dynamics of the α-decay. They are valid for any parity of the initial and the final states.

The existence of relations (6) has a simple physical reason. Consider the α-decay transition $I_\mathrm{i} \to I_\mathrm{f}$ with $I_\mathrm{i} > I_\mathrm{f}$. Suppose that in the initial state only the maximal projection of the nuclear spin is populated, namely $M_\mathrm{i} = I_\mathrm{i}$. Since the projection of the total angular momentum should be conserved in α-decay, the final state consisting of the residual nucleus and the α-particle should also have the total angular momentum projection equal to I_i. Consider emission of α-particles along the z-axis. Then the orbital motion of the α-particle does not give any contribution to the angular-momentum projection. Only the spin of the residual nucleus can contribute. But the maximal possible projection of the final spin is I_f which is less than I_i. Thus it is impossible to satisfy the conservation law for the projection of the total angular momentum. Therefore, the intensity of the α-emission along the nuclear orientation direction should be zero. This leads to the equation

$$W(\theta = 0) = 1 + \sum_{k \neq 0} A_k B_k = 0. \tag{7}$$

We remind that the orientation parameters B_k are defined as follows [11]:

$$B_k = \hat{I}_\mathrm{i} \sum_{M_\mathrm{i}} (-1)^{I_\mathrm{i} - M_\mathrm{i}} (I_\mathrm{i} M_\mathrm{i}, I_\mathrm{i} - M_\mathrm{i} \mid k0) p_{M_\mathrm{i}} \tag{8}$$

where p_{M_i} are populations of the corresponding magnetic substates. In the above-considered case only one substate is populated, $p_{I_\mathrm{i}} = 1$. Thus in this case

$$B_k = \hat{I}_\mathrm{i} (I_\mathrm{i} I_\mathrm{i}, I_\mathrm{i} - I_\mathrm{i} \mid k0). \tag{9}$$

Substituting the orientation (9) in eq. (7) we get relation (6) for the case $M_\mathrm{i} = I_\mathrm{i}$.

As an example, consider the decay $5/2^+ \to 3/2^+$. The selection rules imply that only $L = 2, 4$ contribute and $k_\mathrm{max} = 4$. In this case eq. (6) reduces to

$$1 + 5\sqrt{\frac{1}{14}} A_2 + \sqrt{\frac{3}{14}} A_4 = 0. \tag{10}$$

For another transition $7/2^+ \to 3/2^+$ ($k_\mathrm{max} = 6$) two different equations exist (for $M_\mathrm{i} = 7/2$ and $5/2$):

$$1 + \sqrt{\frac{7}{3}} A_2 + \sqrt{\frac{7}{11}} A_4 + \sqrt{\frac{1}{33}} A_6 = 0 \tag{11}$$

and

$$1 + \sqrt{\frac{1}{21}} A_2 - 13\sqrt{\frac{1}{77}} A_4 - 5\sqrt{\frac{1}{33}} A_6 = 0. \tag{12}$$

In both particular cases considered only one directional distribution coefficient is independent, for example A_2. Others may be found using the above equations. Note that eq. (11) is valid also for the $7/2 \to 5/2$ transitions, and eqs. (10)-(12) are, in fact, independent of the parities of the initial and the final states. Unfortunately, we have not found any published experimental data for the case $I_\mathrm{i} > I_\mathrm{f}$ in order to demonstrate how eq. (6) works. In the majority of papers [6–10] the angular distributions for favoured transitions with $I_\mathrm{i} = I_\mathrm{f}$ have been measured and analyzed.

The basic equation (6) is fulfilled for any spin of the initial state, integer and half-integer. Below we derive another independent equation which is valid only for integer spins. Consider again the sum (4). If the initial spin is integer, one can assume $M_\mathrm{i} = 0$. In this case the right-hand side of the equation turns to zero if $I_\mathrm{i} + I_\mathrm{f} + L$ is an odd number. This leads to the equation

$$1 + \sum_{k \neq 0} (-1)^{I_\mathrm{i}} \hat{I}_\mathrm{i} (I_\mathrm{i}0, I_\mathrm{i}0 \mid k0) A_k = 0. \tag{13}$$

It is clear that eq. (13) exists due to parity conservation in α-decay. As an example, consider the α-transition $2^+ \to 2^-$. In this case the angular distribution is determined by two coefficients, A_2 and A_4. However, only one of the two is independent; the second may be found from the relation (13) which in this case reads

$$1 - \sqrt{\frac{10}{7}} A_2 + 3\sqrt{\frac{2}{7}} A_4 = 0. \tag{14}$$

In fact, this equation is valid for any $2^+ \to I_\mathrm{f}^\pi$ transition if only $I_\mathrm{f} + \pi$ is *odd*. Obviously, eq. (13) cannot be applied in studying the α-transitions from oriented nuclei which are of odd mass and therefore of half-integer spin. However, it may be useful in analysis of α-decay of compound nuclei produced in nuclear reactions. It is well known that the compound nuclei may be strongly oriented [13, 14] and therefore the emitted α-particles may be anisotropic.

Since the derived equations are purely kinematical and do not depend on the dynamics of the α-decay, they are valid for the emission of any zero-spin particles. The found relations may be useful for checking the theoretical calculations or serve as a test of the consistency of experimental data. Finally, we note that similar exact relations exist between coefficients which determine the angular distributions and spin polarization in the decay with emission of spin-(1/2) particles. They have been found and discussed in atomic Auger decay (see [15, 16] and references therein).

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